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Quantum Hydrodynamic Analysis of Electrostatic Waves in Carbon Nanotubes

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Abstract

The linearized quantum hydrodynamic model (QHD) is employed to investigate the dispersion relation of electrostatic waves propagating through Single Walled Carbon Nanotubes (SWCN) subjected to an axial magnetic field. Previously, we theoretically computed the conditions for exciting quantum electrostatic modes in a CNT filled with electrons and exposed to an axial magnetic field, utilizing wavelengths spanning the microwave to visible spectrum range [1, 2]. In this paper, we present the dispersion relations, unveiling the magnetic field's influence on the excitation of quantum acoustical and optical waves. We establish that weak magnetic fields hinder the propagation of quantum acoustical waves, while strong magnetic fields permit it. Additionally, the excitation of quantum optical waves across all magnetic field strengths requires wavelengths from the microwave to visible spectrum. Lastly, we depict the dispersion relations of acoustical and optical modes as a logarithmic function of the magnetic field's base 10. Through a comparative analysis using two models, we reveal intriguing disparities between these two modes. This study might provide a platform to create new finite transverse cross section quantum magnetized plasmas and to devise nanometer dusty plasmas based on the metallic carbon nanotubes in the absence of either a drift or a thermal electronic velocity and their existence could be experimentally examined.

1. Introduction

High temperature and low-density regimes are characteristic of classical plasma. The quantum effects will be manifest when the de Broglie wave lengths of the charge carriers forming the plasma become comparable to its spatial length scale [3-7]. The dynamics of quantum plasmas are mostly treated using the Schrödinger-Poisson model, the Wigner-Poisson formalism, and the quantum hydrodynamic (QHD) equations [8-10]. One of the basic collective phenomena in plasma is electrostatic waves [11-13]. In classical plasmas, in addition to the high frequency Langmuir electron wave and the low frequency ion acoustic wave, one would search for the upper-hybrid and lower-hybrid waves propagating perpendicular to an applied magnetic field [11]. During the past years, a number of studies have been performed on quantum effects in the plasma dynamics. For example, Haas *et al*. derived the dispersion relationship for the two-stream instability and identified a new pure quantum branch [14]. Ren *et al.* have investigated the linear waves in quantum plasmas [15, 16].

Since their discovery in 1992 by Iijima [17-19], Carbon nanotubes (CNTs) of different geometries and functionalities have been considered as possible candidates to devise new generation of nano-waveguides for propagating the electromagnetic waves over a wide range of frequencies. In special, surface excitation modes are one of the most fascinating aspects in CNTs. Many experimental realizations [20, 21] and theoretical works [1, 2, 22-24] have been done to describe the high-frequency excitations (electron oscillations) in these systems. Wei and Wang, using the QHD model, have studied the quantum ion-acoustic waves in single-walled carbon nanotubes (SWCNTs) [25].

Additionally, the effects of a static magnetic field on collective modes in CNTs have been investigated by several authors. Fetter, based on a simple hydrodynamic model, obtained an acoustical branch in addition to the optical [26]. Using the tight-binding model, Shyu *et al* studied the magneto plasmon of SWCNTs [27]. Chiu *et al* studied the low-frequency single-particle and collective excitations of SWCNTs in the presence of a magnetic field [28, 29]. Abdikian *et al*. by exploiting the QHD model and Maxwell's equations have tried to study the electrostatic waves in carbon nanotube which filled by completely electron gas with axial magnetic field [24]. They have suggested one may use the spectra located between macro-wave and visible to excite these waves.

In the present study, we focus on the propagation of linear electrostatic waves by using the last method with frequency near the electron plasma frequency, for magnetized partially cold quantum plasma confined on the surface of a CNT in the presence of a static axial magnetic field. We utilize the usual QHD to derive the corresponding expressions identifying the collective excitations.

The paper is organized as follow. In the next section we bring our main equations. In section III our results are discussed. Section IV is supplied by our conclusion.

2. THE BAIC DIFFERENTIAL EQUATIONS

We consider a quantum electron fluid moving on the surface of an infinitely long and infinitesimally thin SWMCNT with radius R subject to a finite static axial magnetic field$B_0\hat{\boldsymbol{z}}$. The dynamics of the electron is governed by the magneto-hydrodynamical equations [30, 31]. Two types of propagating plasma and cyclotron type waves will be analyzed in the electrostatic approximation [24]. For these high frequency waves, the positive ions provide a uniform stationary background of positive charge. The dynamics of the electron is governed by the magneto-hydrodynamical equations [32]

$$\nabla^2\Phi = \frac{e}{m}(n - n_0), \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{v}) = 0, \tag{2}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \frac{e}{m}[\nabla\Phi + \boldsymbol{v} \times \boldsymbol{B}] - \frac{\nabla P}{mn} + \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right), \tag{3}$$

where $n_0 = n_{0e}$ represents the equilibrium electron number density which is uniform and constant through the plasma, $\hbar$ is the Planck constant divided by 2π, P is the pressure and m is the electron mass. If we set $\hbar$ equal to zero, we obtain the classical hydrodynamic equations. Here, we neglected the motion of ions and assume that the ion number density $n_i = n_0$. Also, we suppose cold plasma where the thermal pressure$p$ is ignored. Choosing the cylindrical polar coordinates $X(r, \varphi, z)$ for an arbitrary point in the space, one would obtain the linearized aforementioned equations as follow

$$\nabla^2 \Phi_1 = \frac{e n_1}{m_0}, \tag{4}$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot (\boldsymbol{v}_1) = 0, \tag{5}$$

$$\frac{\partial \boldsymbol{v}_1}{\partial t} = \frac{e}{m_0} \nabla \Phi_1 - \omega_c \boldsymbol{v}_1 \times \hat{\boldsymbol{z}} + \frac{\hbar^2}{4 m_0 n_0} \nabla (\nabla^2 n_1), \tag{6}$$

Here $n_1, \boldsymbol{v}_1$and Φ_1 are the perturbed parts of the electron density, the electron fluid velocity and the potential associated with a small electrostatic perturbation and $\omega_c = eB_0/m$ is the electron cyclotron frequency. The quantum term (Bohm potential) is approximated by,

$$\frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = \frac{1}{2n_0} \nabla^2 n_1$$

Which holds for linear a perturbation $n_1 << n_0$ [32]. Combining the above system of equations for $n_1, \boldsymbol{v}_1$, and Φ_1, one may reduce to a single differential equation for Φ_1. By eliminating $n_1$from Eqs. (4) and (5) one would obtain

$$e n_0 \nabla \cdot \boldsymbol{v}_1 + \varepsilon_0 \nabla^2 \frac{\partial \Phi_1}{\partial t} = 0. \tag{7}$$

For the z component of Eq. (6), the z component of velocity is eliminated from Eq. (6) as,

$$\frac{m\omega_p^2}{e} \frac{\partial}{\partial t} \nabla_\perp \cdot \boldsymbol{v}_{1\perp} + \omega_p^2 \frac{\partial^2 \Phi_1}{\partial z^2} + \lambda_{qe}^4 \omega_p^2 \frac{\partial^2}{\partial z^2} (\nabla^4 \Phi_1) + \nabla^2 \frac{\partial^2 \Phi_1}{\partial t^2} = 0, \tag{8}$$

where $\omega_p = \left(\frac{e^2 n_0}{m\varepsilon_0}\right)^{1/2}$ is the plasma frequency and $\lambda_{qe}^4 = \hbar^2 / \left(4m^2\omega_p^2\right)$ is the quantum wavelength of electron [1]. Taking the divergence of the transvers component in Eq. (6) yields

$$\frac{\partial}{\partial t} \nabla_\perp \cdot \boldsymbol{v}_{1\perp} = \frac{e}{m} \nabla_\perp^2 \Phi_1 - \omega_c \hat{\boldsymbol{z}} \cdot (\nabla \times \boldsymbol{v}_{1\perp}) + \frac{e}{m} \lambda_{qe}^4 \nabla_\perp^2 (\nabla^4 \Phi_1). \tag{9}$$

Also, taking the curl of the transvers component in Eq. (6) for its$z$ component results in

$$\frac{\partial}{\partial t} \hat{\boldsymbol{z}} \cdot (\nabla \times \boldsymbol{v}_{1\perp}) = \omega_c \nabla_\perp \cdot \boldsymbol{v}_{1\perp}, \tag{10}$$

Now, eliminating $\hat{\boldsymbol{z}} \cdot (\nabla \times \boldsymbol{v}_{1\perp})$ in Eqs. (9) and (10), one would find the following expression for transvers component as

$$\left(\omega_c^2 + \frac{\partial^2}{\partial t^2}\right) \nabla_\perp \cdot \boldsymbol{v}_{1\perp} = \frac{e}{m} \nabla_\perp^2 \frac{\partial \Phi_1}{\partial t} + \frac{e}{m} \lambda_{qe}^4 \nabla_\perp^2 \frac{\partial}{\partial t} (\nabla^4 \Phi_1), \tag{11}$$

Combining Eqs. (8) and (11), one may obtain the equation governing the electrostatic waves propagating along the nanotube axis in an axial magnetic field

$$\left(\omega_c^2+\frac{\partial^2}{\partial t^2}\right)\left(\omega_p^2+\lambda_{qe}^4\omega_p^2\nabla^4+\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2\Phi_1}{\partial z^2}+\left(\omega_p^2+\omega_c^2+\lambda_{qe}^4\omega_p^2\nabla^4+\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2}{\partial t^2}\nabla_\perp^2\Phi_1=0 \quad (12)$$

In the next part, we exploit Eq. (12) to derive the dispersion relations of the electrostatic waves. If the potential is chosen to be of the form $\Phi_1=\widehat{\Phi}_1\,exp[i(k_z z-\omega t)]$, where k_z and ω are the wavenumber and the angular frequency, respectively, with $\widehat{\Phi}_1$ as a function of the transverse coordinate, then the dispersion relation for electrostatic waves in completely filled SWCNT plasma taken by Eq. (12) as well as taken in [24]

$$\omega^4-\omega^2\left(\omega_p^2+\omega_c^2+\lambda_{qe}^4\omega_p^2k^4\right)+\frac{k_z^2\omega_c^2}{k^2}\left(\omega_p^2+\lambda_{qe}^4\omega_p^2k^4\right)=0, \quad (13)$$

here $k_\perp=p_{l\nu}/R$ which $p_{l\nu}$ is the ν^{th} zero of the J_l Bessel function and $k=\sqrt{k_\perp^2+k_z^2}$.

3. RESULTS AND DISCUSION

By using the Eq. (13) derived in before section, one may be employed to estimate dispersion curves for two branches of quantum electrostatic waves [24]. It is convenient to classify the two types of electrostatic modes as the lower-frequency if $\omega>\omega_p$ and the higher-frequency if $\omega>\omega_c$ [11]. The lower-frequency solution of Eq. (13) yields the quantum acoustical modes. This type of mode is zero at $k=0$, increases with increasing k, and approach an asymptotic limit (ω_p or ω_c, whichever is smaller) as k approaches infinity. On the other hands, the higher-frequency solution of Eq. (13) yields the quantum optical modes in SWCNTs. This electrostatic approximation predicts that the frequencies of the optical modes are equal to the upper hydride frequency ($\omega_H=\sqrt{\omega_p^2+\omega_c^2}$) at $k=0$, decrease with increasing k, and approach an asymptotic limit (ω_c or ω_p, whichever is larger) as k approaches infinity.

The normalized acoustical frequency ω/ω_p is shown as a function of the logarithm of the normalized wavenumber, to the base 10 in Figure 1. This Figure emphasis that the all curves, in the limit of $k_z\to 0$, tending to be zero, and the classical and QHD models are accordance with each other for $\omega_c/\omega_p<1$ as wave number increasing. On the other words, the quantum affects (i.e. for $\omega_c/\omega_p<1$) is not observable, with the weak magnetic field, however with the strong magnetic field (i.e. for $\omega_c/\omega_p>1$) these effects is found.

In Figure 2 the normalized optical frequency ω/ω_p is shown as a function of the logarithm of the normalized wavenumber, to the base 10 which indicates the region of both modes with the same values is restricted to $10^{-2}<R_bk_z<2\times10^5$ (or $-12<log(R_bk_z)<-6$) for the all magnetic field values (the weak and strong values); so the optical quantum mode calculated by both models collapse together in this region. On the other hands, this Figure shows that the optical quantum effects by QHD model for big wave number limitations with the any magnetic field (i.e., for all ω_c/ω_p) values is observable, however, in classical approach, when the wave number increases, the optical plasmon modes tend to asymptotically be the constant.

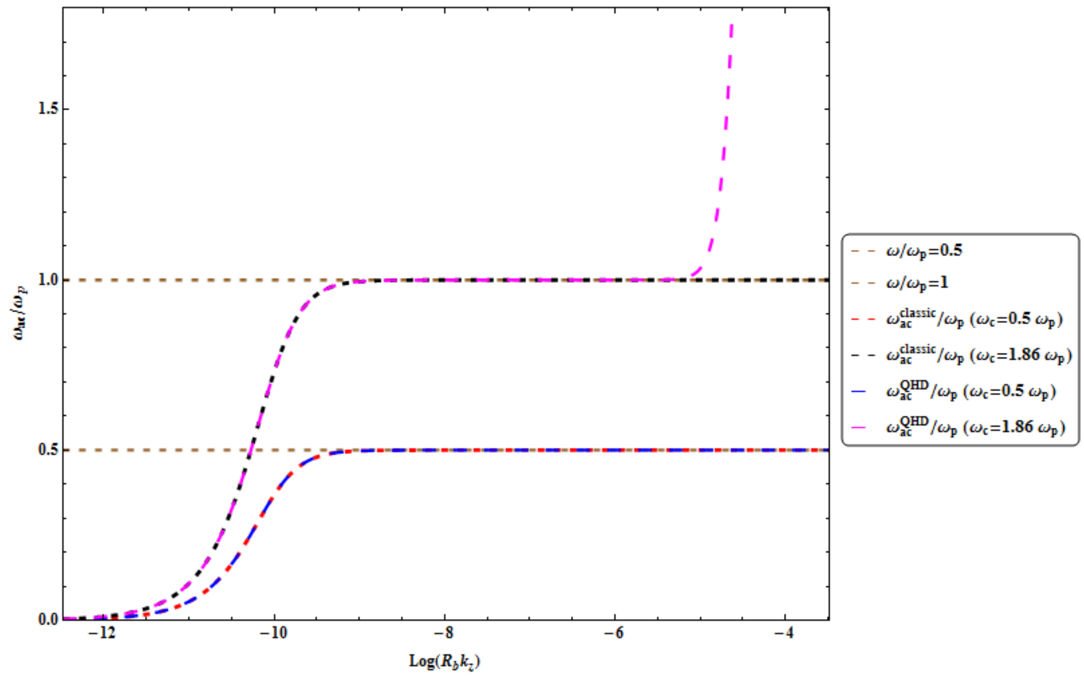

Fig. 1. Scheme of the normalized acoustical frequency ω/ω_p as a function of the logarithm of the normalized wavenumber.

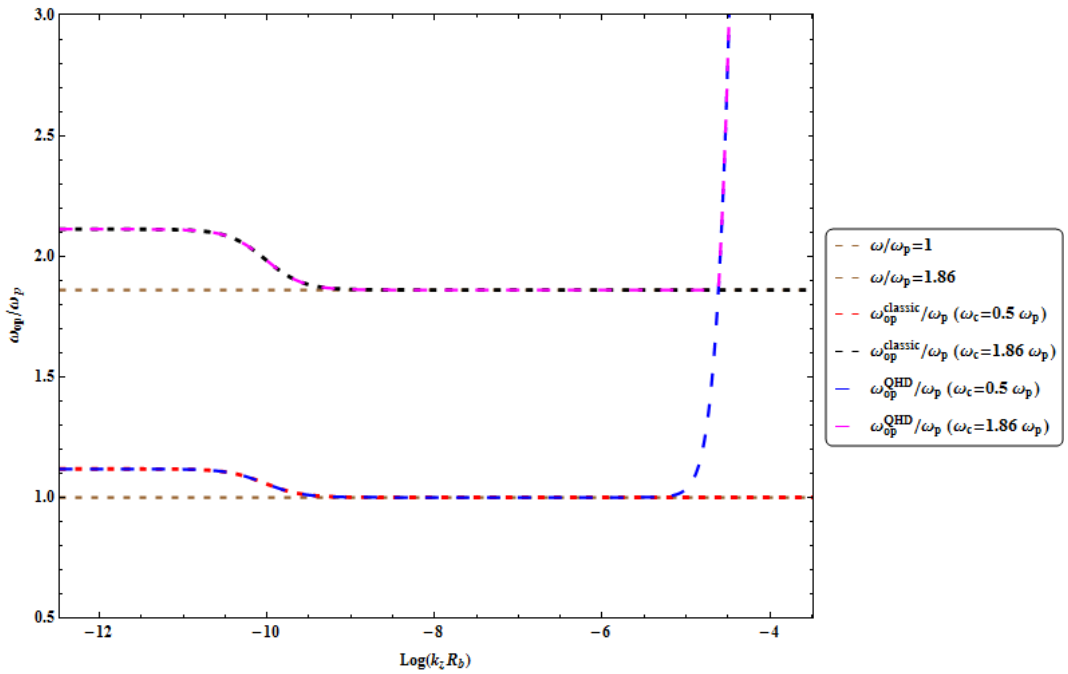

Fig. 2. Schematic of the normalized optical frequency ω/ω_p as a function of the logarithm of the normalized wavenumber.

In Figures 3 and 4 the normalized acoustical and optical frequency ω/ω_p is shown as a function of the logarithm of the normalized cyclotron frequency ω_c/ω_p to the base 10. The first indicates the acoustical mode by QHD model (for the weak and strong magnetic values) is different from classical approach and is located approximately tiny above it. But for optical modes have the same values for the all-magnetic values (the weak and strong) in two methods.

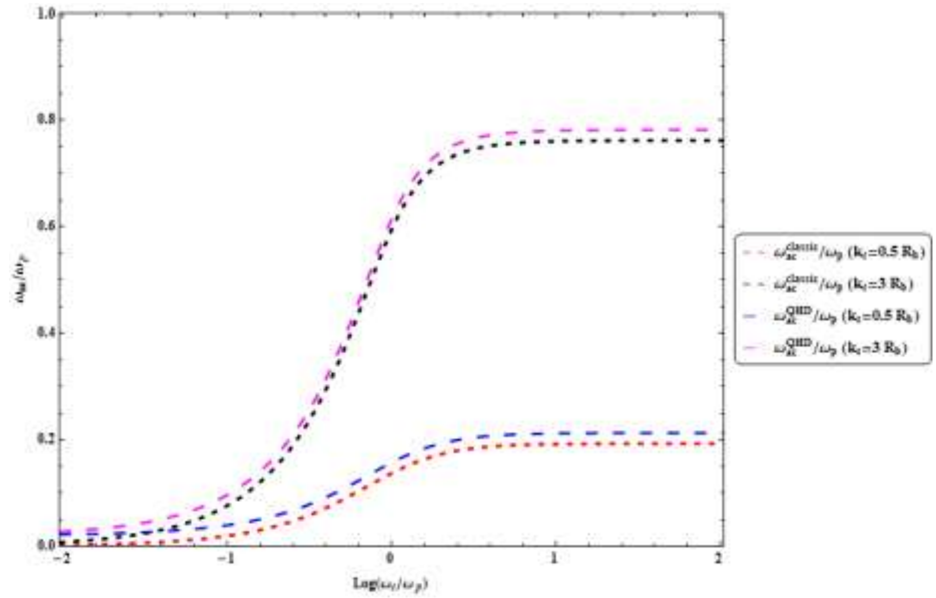

Fig. 3. The normalized acoustical frequency ω/ω_p as a function of the logarithm of the normalized cyclotron frequency ω_c/ω_p.

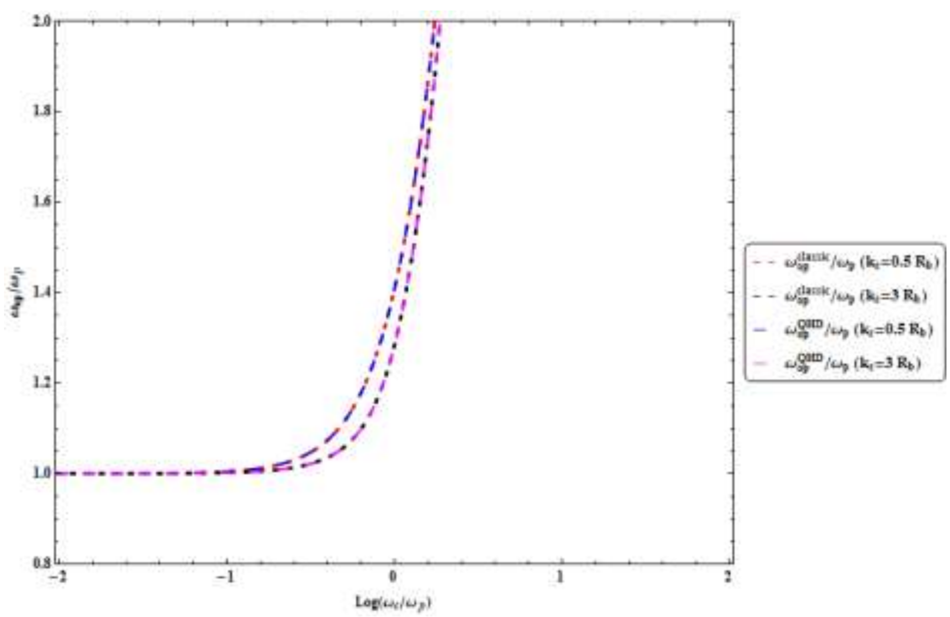

Fig. 4. The normalized optical frequency ω/ω_p as a function of the logarithm of the normalized cyclotron frequency ω_c/ω_p.

4. Conclusion

In this study, based on the linearized quantum hydrodynamic model, we have derived an analytical expression for the dispersion of electrostatic waves in uniform and cold magnetized plasma confined in single-walled carbon nanotubes. In addition, by some drawing the dispersion relations, we have computed how magnetic field can effect on excitation of quantum acoustical and optical waves. We have found that for weak magnetic fields the quantum acoustical waves can't propagate however for strong magnetic field it is possible. To excite this mode, one may use wavelength between microwave and visible spectrum. Also, for all magnetic fields to excite quantum optical waves, one may use wavelengths as before. Besides, by depicted the dispersion relations of acoustical and optical modes as a function of the logarithm of the magnetic field to the base 10, we have found that for the small value of wavenumber even for sufficiently large magnetic field both optical waves from two models have equal values, but the value of acoustical waves by the QHD model is a bit bigger than of the classical approach.

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